Course guide
34966 - VD - Differentiable Manifolds

Last modified: 11/06/2026

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
981 - CRM - Mathematical Research Centre.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2026 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: EVA MIRANDA GALCERÁN

Others: Segon quadrimestre:
EVA MIRANDA GALCERÁN - A

PRIOR SKILLS

Recommended background:

Students are expected to have some previous background in algebra, calculus, topology, differential equations, and calculus on manifolds.

This is not an introductory course. Participants should ideally have already attended courses in differential geometry and smooth manifolds. However, students who are unsure whether they meet the recommended background are warmly encouraged to contact the lecturers to discuss their situation.

Students with less experience in some of these topics may still be able to follow the course successfully, provided they are willing to fill any gaps through individual study during the course and/or by reading recommended material before the course begins.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Theory and tutorial sessions will be used to present and develop the course content.Theory and tutorial sessions will be used to present and develop the course content. Throughout the course, students will be assigned problem sets as homework. In addition, a series of special lectures will explore complementary aspects of the theory, designed to inspire students to engage with cutting-edge research and to set them on their way toward writing their essay.

LEARNING OBJECTIVES OF THE SUBJECT

The subject focuses on some of the fundamental topics of differential geometry and its applications to different areas including mathematical physics and Dynamical systems.

By the end of the course, students should be able to:
- understand all the ideas developed along the course.
- apply the studied concepts to other areas of pure mathematics, physics and engineering.
- integrate in a research group on these kinds of topics and their applications.
- search and understand the scientific literature on the subject.
- write and present an essay on mathematics.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Complements in Differential Geometry

Description:
Brief survey of manifold theory and differential geometry including differential forms.
We also plan to talk about differentiable distributions and study its integration via the theorem of Frobenius. This will lead us to introducing several examples of foliations.

Full-or-part-time: 30h 30m
Theory classes: 8h
Self study : 22h 30m

Basic notions on De Rham Cohomology

Description:
We define De Rham cohomology and compare it to other cohomologies. (Depending on the preliminary knowledge of the students, this chapter may be considered as an APPENDIX)

Full-or-part-time: 16h 30m
Theory classes: 4h
Self study : 12h 30m

Introduction to Differential Topology

Description:
We present a brief introduction to the theory of Differential Topology which includes basic notions in transversality, singularity theory and Morse theory.

Full-or-part-time: 28h
Theory classes: 11h
Self study : 17h

Introduction to Lie theory

Description:
A Lie group is a group endowed with a smooth manifold structure which is compatible with the group operation. In this chapter we provide an
introduction to the main aspects of the theory of Lie groups and Lie algebras taking matrix Lie groups as starting point.

Full-or-part-time: 32h
Theory classes: 11h
Self study : 21h

Lie group actions on smooth manifolds

Description:
We study Lie group actions on smooth manifolds and relate both geometries via the notions of isotropy group and orbit.

Full-or-part-time: 18h 30m
Theory classes: 6h
Self study : 12h 30m

Introduction to Symplectic and Poisson Geometry

Description:
We provide a comprehensive introduction to symplectic and Poisson manifolds with special focus on examples. Starting with symplectic manifolds, we will explain Moser's trick and some applications to normal form theorems such as the Darboux theorem and the classification of symplectic surfaces. We introduce the notion of Hamiltonian vector field, symplectic vector field and Hamiltonian System. Special attention will be given to examples provided by the realm of integrable systems. In particular the action-angle theorem of Arnold-Liouville will be presented and the notion of moment map and Hamiltonian group action. We end the chapter introducing other geometries: that of contact geometry and Poisson geometry discussing the most recent progress on open conjectures in the topic. We discuss the notion of symplectic foliation and present the recent advances in contact, symplectic and Poisson Geometry.

Full-or-part-time: 62h
Theory classes: 20h
Self study : 42h

GRADING SYSTEM

There will be exam(s) contributing 30% to the final grade, and an essay contributing a further 40%. Students will choose, together with the lecturers, a topic that complements or advances the material covered during the course, according to their mathematical interests. The remaining 30% will be based on problem solving, evaluated through selected assignments from the regular problem sets. Students will have to present the detailed solution of some of these problems in written form.

EXAMINATION RULES.

The final grade awarded to each student will be computed as follows:

40% exam(s) + 40% essay + 20% problem solving.

The grade "exam(s)" includes the one of final exam but may also include other examination material such as ATENEA questionnaires or take-home exercises. The choices and number
of exams will depend on several factors including the ratio of in-person versus online teaching.

BIBLIOGRAPHY

Basic:
- Bröcker, T.; Jänich, K. Introduction to differential topology. Cambridge University Press, 1982. ISBN 9780521284707.
- Cannas da Silva, Anna. Lectures on symplectic geometry [on line]. Springer-Verlag, 2008 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-45330-7. ISBN 354045330X.
- Duistermaat, J. J.; Kolk, Johan A.C. Lie groups. Berlin: Springer-Verlag, 2000. ISBN 3540152938.
- Fegan, H.D. Introduction to compact lie groups. World Scientific, 1991. ISBN 9810236867.
- Guillemin, V.; Sternberg, Shlomo. Symplectic techniques in physics. Repr. with corrections. Cambridge [etc.]: Cambridge University Press, 1990. ISBN 9780521389907.
- Guillemin, Victor; Pollack, Alan. Differential topology. Reprint of the 1974 original. AMS Chelsea Publishing, 2014. ISBN 9780821851937.
- Knauf, A. Mathematical physics : classical mechanics. Springer, 2018. ISBN 9783662557747.
- Lee, John M. Introduction to smooth manifolds [on line]. New York: Springer-Verlag, 2003 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4419-9982-5. ISBN 0387954481.
- Milnor, John W. Topology from the differentiable viewpoint. Rev. ed. Princenton: Princeton University Press, 1997. ISBN 9780691048338.
- Munkres, J.R. Elementary differential topology. ISBN 9780691090931.
- Tu, Loring W. An Introduction to manifolds [on line]. 2nd ed. New York: Springer, 2011 [Consultation: 10/07/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-1-4419-7400-6. ISBN 9781441973993.
- Warner, Frank W. Foundations of differentiable manifolds and Lie groups. New York [etc.]: Springer, cop. 1971. ISBN 0387908943.
- Geiges, Hansjörg. The Geometry of celestial mechanics. Cambridge: Cambridge University Press, 2016. ISBN 9781107125407.
- Geiges, Hansjörg. An Introduction to contact topology. Cambridge: Cambridge University Press, 2008. ISBN 9780521865852.
- Crainic, Marius; Fernandes, Rui Loja; Marcut, Ioan. Lectures on Poisson geometry. Providence, Rhode Island: American Mathematical Society, [2021]. ISBN 9781470464301.
- McDuff, Dusa; Salamon, D. Introduction to symplectic topology. Third edition. Oxford, UK: Oxford University Press, 2017. ISBN 9780198794899.

Complementary:
- Bott, Raoul; Tu, Loring W. Differential forms in algebraic topology. New York: Springer-Verlag, 1982. ISBN 0387906134.
- Audin, Michèle. Torus actions on symplectic manifolds. 2nd ed. Birkhäuser, 2004. ISBN 3764321768.
- Warner, Frank W. Foundations of differentiable manifolds and lie groups. New York, NY [etc.]: Springer-Verlag, cop. 1983. ISBN 0387908943.
- Olver, Peter J. Applications of Lie groups to differential equations. New York: Springer-Verlag, 1986. ISBN 0387940073.
- Nakahara, Mikio. Geometry, topology, and physics. 2nd ed. New York [etc.]: Taylor & Francis, cop. 2003. ISBN 0750306068.
- Audin, Michèle; Damian, Mihai. Morse theory and Floer homology. New York: Springer, [2014]. ISBN 9781447154952.
- Gallier, Jean; Quaintance, Jocelyn. Notes on differential geometry and Lie groups [on line]. University of Pennsylvania, 2016 [Consultation: 10/07/2023]. Available on: http://www.cis.upenn.edu/~jean/gbooks/manif.html.

RESOURCES

Other resources:
Notes on the Geometry and Dynamics of singular symplectic manifolds (notes on the FSMP course by Eva Miranda)

Course on youtube by Professor Eva Miranda on Lie group actions
https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view_as=subscriber />Course on youtube by Professor Eva Miranda

on Symplectic and Poisson Geometry
https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view_as=subscriber