

Course guide

34961 - QQMDS - Quantitative and Qualitative Methods in Dynamical Systems

Last modified: 03/11/2025

Unit in charge:	School of Mathematics and Statistics	
Teaching unit:	749 - MAT - Department of Mathematics.	
Degree:	MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).	
Academic year: 2026	ECTS Credits: 7.5	Languages: English

LECTURER

Coordinating lecturer:	PAU MARTIN DE LA TORRE
Others:	Primer quadrimestre: PAU MARTIN DE LA TORRE - A MARIA TERESA MARTINEZ-SEARA ALONSO - A

PRIOR SKILLS

Good knowledge of calculus, algebra and differential equations. It is strongly recommended a good understanding of the basic theory of ordinary differential equations as well as a basic knowledge of dynamical systems from a local point of view.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

We do not distinguish theoretical and practical classes. Some results about modern theory in Dynamical systems are presented in class. The main idea is to give basic knowledge and useful tools in the study of a dynamical system from both quantitative and qualitative points of view. We will stress the relation between different kind of systems and we will mainly focus in the use of perturbative techniques to study a dynamical system globally.

LEARNING OBJECTIVES OF THE SUBJECT

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Invariant objects in Dynamical Systems

Description:

Continuous and discrete Dynamical Systems.
Poincaré map.
Local behaviour of hyperbolic invariant objects. Conjugation.
Invariant manifolds.

Full-or-part-time: 10h

Theory classes: 10h

Normal forms

Description:

Poincaré-Dulac normal forms. Convergence: Poincaré and Siegel domains.

Full-or-part-time: 10h

Theory classes: 10h

Perturbation theory in Dynamical Systems

Description:

Classic perturbation theory. Averaging theory. Perturbed homoclinic orbits in the plane. Melnikov method. Singular perturbation theory.

Full-or-part-time: 15h

Theory classes: 15h



Bifurcations

Description:

Local bifurcations for planar vector fields and real maps. Saddle node and Hopf bifurcations.

Full-or-part-time: 10h

Theory classes: 10h

Homoclinic points and chaotic Dynamics

Description:

Smale horseshoe. Homoclinic points and bifurcations. Hyperbolic sets and transversal homoclinic points. Dynamical systems with chaotic dynamics. Newhouse phenomenon.

Full-or-part-time: 10h

Theory classes: 10h

Non-smooth systems

Description:

Introduction to non-smooth differential equations. Definition and motivating examples. Filipov's convention.

Full-or-part-time: 5h

Theory classes: 5h

GRADING SYSTEM

The students have to do some problems (60%) and a research work (25%). There will be also a final exam covering on the theoretical part of the subject (15%). On the other hand they will attend the winter courses "Recent trends in non-linear science" and produce a document about them.

EXAMINATION RULES.

There will be a final exam covering the theoretical part of the course.

BIBLIOGRAPHY

Basic:

- Arrowsmith, D. K; Place, C. M. An Introduction to dynamical systems. Cambridge [England] ; New York: Cambridge University Press, 1990. ISBN 0-521-30362-1.
- Guckenheimer, John; Holmes, Philip. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. New York, NY [etc.]: Springer-Verlag, 1983. ISBN 0-387-90819-6.
- Katok, Anatole; Hasselblatt, Boris. Introduction to the modern theory of dynamical systems. Cambridge [etc.]: Cambridge University Press, 1995. ISBN 0521341876.
- Hasselblatt, Boris; Katok, A. B. A First course in dynamics : with a panorama of recent developments. Cambridge [etc.]: Cambridge University Press, 2003. ISBN 0521583047.
- Hirsch, Morris W.; Smale, Stephen; Devaney, Robert L. Differential equations, dynamical systems, and an introduction to chaos. 3rd Edition. Oxford: Elsevier, 2013. ISBN 9780123820105.