UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Course guide
34953 - NT - Number Theory

Last modified: 03/11/2025

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2026 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: JOAN CARLES LARIO LOYO

Others: Primer quadrimestre:
JORDI GUARDIA RUBIES - A
JOAN CARLES LARIO LOYO - A

PRIOR SKILLS

Basic knowledge of algebraic structures: groups, rings and fields.

REQUIREMENTS

Basic material covered in any standard courses on arithmetic, group theory and Galois theory.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Most of the lectures will take place on the blackboard (replaced by online lessons if necessary), explaining carefully the contents of the course and providing as much explicit examples, exercises and applications as possible. The students will be encouraged to consult suitable references and to discuss between them and with the professor in order to achieve a good understanding of the material.

LEARNING OBJECTIVES OF THE SUBJECT

1) Algebraic number theory.
2) Number theory in function fields.
2) Cyclotomic theory.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Algebraic Number Theory

Description:
Chapter 1: p-adic Arithmetic

1.1. Introduction to p-adic Numbers

Motivation: Why study p-adics?
Construction of â□¤â□□ and â□□â□□ via p-adic expansions.
Examples and Software.

1.2. Arithmetic in â□□â□□

Basic operations (addition, multiplication, inverses).
Algebraic structure: â□¤â□□ as a DIP
Hensel's Lemma
Multiplicative structure: Units in â□¤â□□ and p-adic exponents.

Chapter 2: Valuations and Local Fields

2.1. Absolute Values and Completions

p-adic valuations and the ultrametric inequality.
Completing â□□ to get â□□â□□.
Ostrowski's Theorem.

2.2. Topology of â□□â□□

Metric space properties (totally disconnected, compactness of â□¤â□□).
Comparison with real numbers (â□□ vs. â□□â□□).

2.3. Local Fields

Definition of local fields (finite extensions of â□□â□□ or ð□□½â□□((t))).
Structure of the valuation ring and residue field.

Chapter 3: Extensions of â□□â□□

3.1. Finite Extensions

Extending valuations (ramification index, inertia degree).
Norm and trace in p-adic fields.
Unramified and totally ramified extensions.
Krasner's Lemma.
Galois groups of p-adic extensions.

3.2. Algebraic Closures and â□□â□□

The algebraic closcar of â□□â□□ and its completion â□□â□□.

Chapter 4: Applications

4.1. Hasse-Minkowski Theorem

Quadratic forms over â□□â□□.
Local-global principle for â□□ (counterexamples and applications).

4.2. p-adic Analysis

p-adic integration (Haar measure, Volkenborn integral).
Zeta functions and p-adic L-functions (brief intro).

4.3. p-adic Galois Representations

Motivation from number theory (Fermat’s Last Theorem).
Tate modules and â□□â□□-linear representations.

Full-or-part-time: 10h
Theory classes: 10h

Number Theory in function fields

Description:
Functions fields over finite fields
Carlitz polynomials
Carlitz extensions

Full-or-part-time: 10h
Theory classes: 10h

Cyclotomic Theory

Description:
Cyclotomic integers
Cyclotomic units
Unique factorization
Class numbers
Galois action
Kronecker-Weber theorem
Regular polygons
Fermat equation
Quadratic reciprocity

Carlitz modules
Galois action
Carlitz-Hayes theorem
Cyclotomic and Carlitz analogies
Quadratic reciprocity
Drinfeld modules

Full-or-part-time: 40h
Theory classes: 40h

GRADING SYSTEM

There will be a final exam.

EXAMINATION RULES.

Solved exercises and works must be delivered according to schedule. Final exam, if necessary.

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BIBLIOGRAPHY

Basic:
- Serre, Jean Pierre. A Course in arithmetic. New York, 1973. ISBN 0387900403.
- Gouvêa, Fernando Q. P-adic Numbers: an introduction. Berlin, 1993.
- Neukirch, J. Algebraic number theory. Berlin, 1999. ISBN 3540653996.
- Koblitz, Neal. P-adic numbers, p-adic analysis, and zeta-functions. New York, 1996. ISBN 0387960171.
- Cassels, J.W.S. Local fields. Cambridge, 1986. ISBN 0521304849.
- Robert, A. A Course in p-adic analysis. New York, 2000. ISBN 0387986693.

RESOURCES

Computer material:
- SAGE. Mathematical Software
- Matlab. Mathematical software